Contract Design Under Uncertainty

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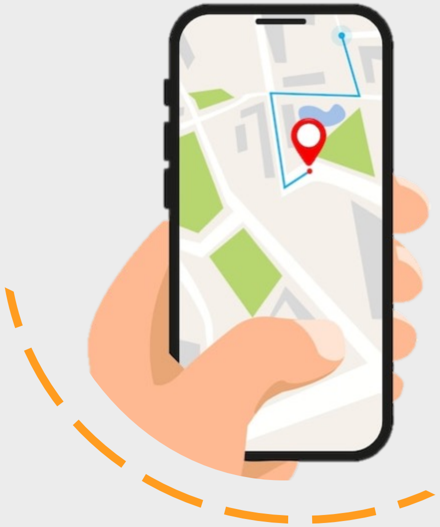
Modern Algorithms in Society

- Interactions with **self-interested** individuals
- Challenges beyond **computational** tractability
- Take **incentives** into account



Social Media Marketing

- **Paid Ads** - in the social platform feed
- **Influencer Marketing** - a brand hires popular users

Paid Ads

- Algorithmic **auction** - advertisers bid → allocation and payments

Input —hidden type→ **Algorithm** → Output

- ✓ Computational Complexity
- ✓ Achieving optimality
- ✓ **Incentivize truthfulness**

Influencer Marketing

- The algorithm determines a **contract** to incentivize effort

Input → **Algorithm** → Output

- ✓ Computational Complexity
- ✓ Achieving optimality
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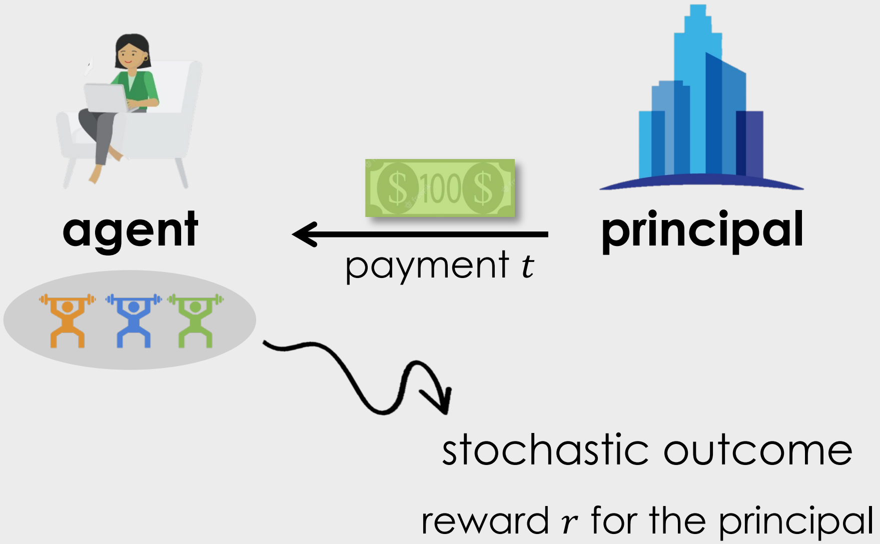
The Principal-Agent Problem [GH83]

- **Moral hazard** - the agent's actions cannot be observed
- **Objective:** a contract maximizing expected **rewards minus payment** $\mathbb{E}[r - t]$



Research Agenda

Generalizations of the classic model:

- **Personalization** for participants from diverse population (with **types**)
- **Multilateral** contracts involving multiple principals or agents
- The need for **simple** contracts

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Input —hidden type→ **Algorithm** —hidden action→ Output

- ✓ Computational Complexity
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- ✓ **Incentivize effort**
- ✓ **Incentivize truthfulness**

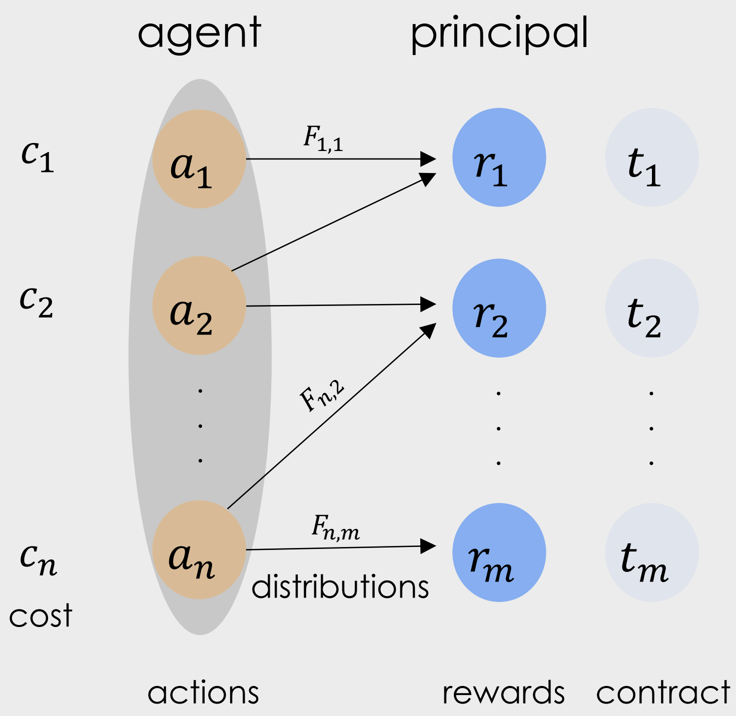
Outline

- **Single-parameter** model of types [ADT.C. EC'21]
  - Motivated by single-parameter auction design
- **Characterization** of the design space [ADT.C. EC'21]
- Counter-intuitive and **undesirable** properties of **optimal contracts** [ADLT.C. EC'23]
- **Linear contracts** (aka commission-based) are near-optimal [ADLT.C. EC'23]

Recent works on contracts with types. Myerson (1982), Guruganesh et al. (2021), Castiglioni et al. (2021), Gottlieb and Moreira (2022), Casto-Pire et al. (2022).

The Model

- The classic principal-agent model by Grossman-Hart (1983)

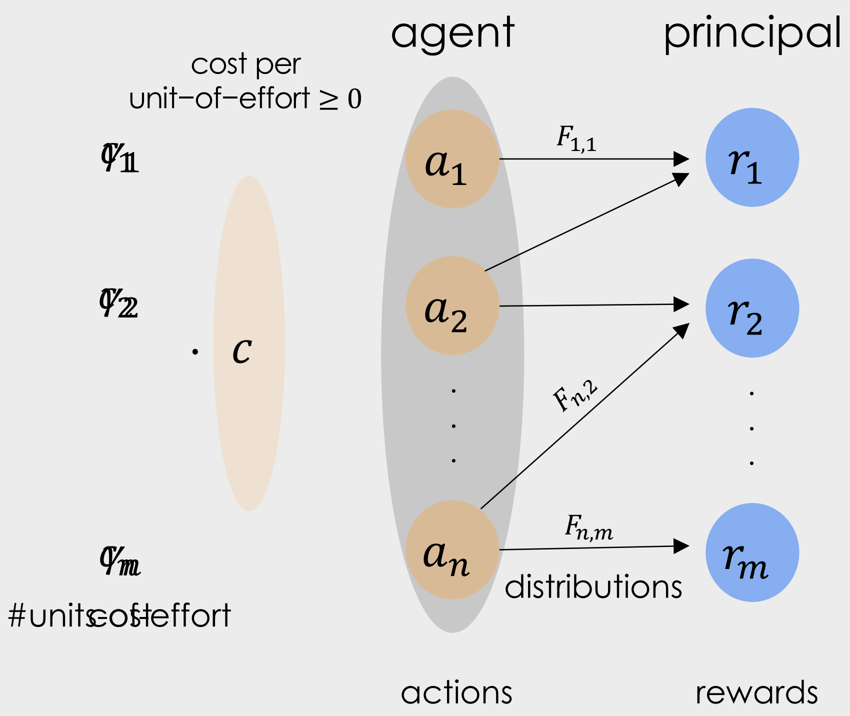


Notation. $T_i = \mathbb{E}_{j \sim F_i}[t_j]$, $R_i = \mathbb{E}_{j \sim F_i}[r_j]$

- Agent's **action i^*** maximizes $T_i - c_i$
- Principal's **revenue** $R_{i^*} - T_{i^*}$

The Model

- Type is drawn from G with density g supported on $C = [\underline{c}, \bar{c}]$



The Model

- A **contract** (x, t) consists of two mappings:
  - **Payment rule** $t: C \to \mathbb{R}^m$ from types to a **paymet** scheme
  - **Allocation rule** $x: C \to [n]$ from types to an **action** recommendation

Notation. $T_i^c = \mathbb{E}_{j \sim F_i}[t(c)_j]$

- **Agent** c report $\hat{c}$ and action $i^*(c)$ maximize $T_{i^*(c)}^{\hat{c}} - \gamma_{i^*(c)} c$
- A contract (x, t) is **incentive compatible (IC)** if $c = \hat{c}$ and $x(c) = i^*(c)$
- **Principal's contract** maximizes $\mathbb{E}_{c \sim G}[R_{x(c)} - T_{x(c)}^c]$ s.t. IC
- **Welfare** sum of utilities $\mathbb{E}_{c \sim G}[R_{i^*(c)} - \gamma_{i^*(c)} c]$

Design Space Characterization

Definition. Allocation rule x is **implementable** if exists payment rule t s.t. (x, t) is IC

Q. What do implementable allocation rules look like?

pure hidden-action characterization

pure hidden-type characterization

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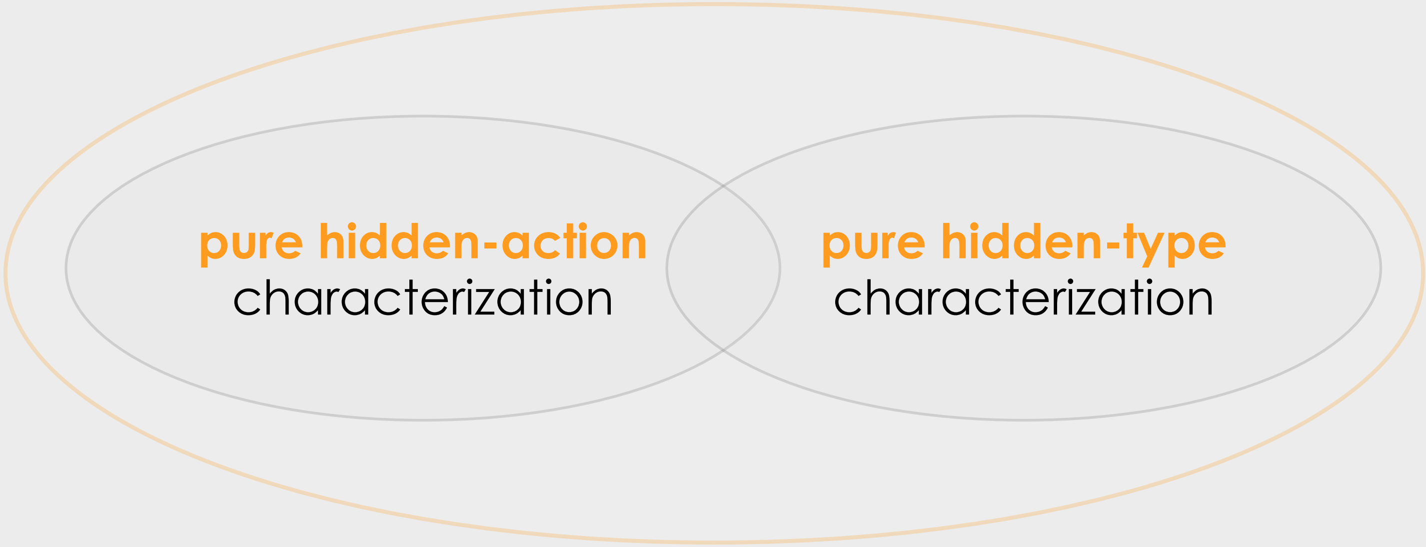
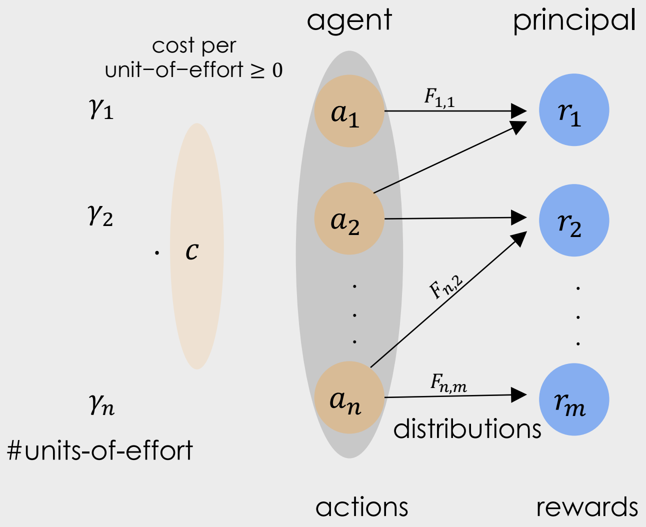


Design Space Characterization

Pure hidden action

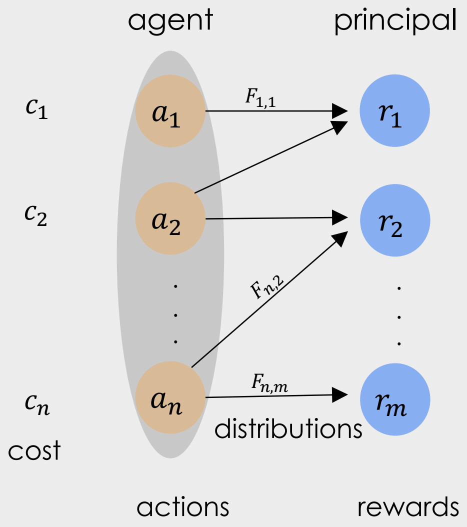


Design Space Characterization

Pure hidden action

- x is **implementable** $\Leftrightarrow$ exists no **deviation scheme** λ_k s.t. (1) dominant distribution $\sum_k \lambda_k F_k \geq F_x$

  (2) strictly lower cost $\sum_k \lambda_k c_k < c_x$



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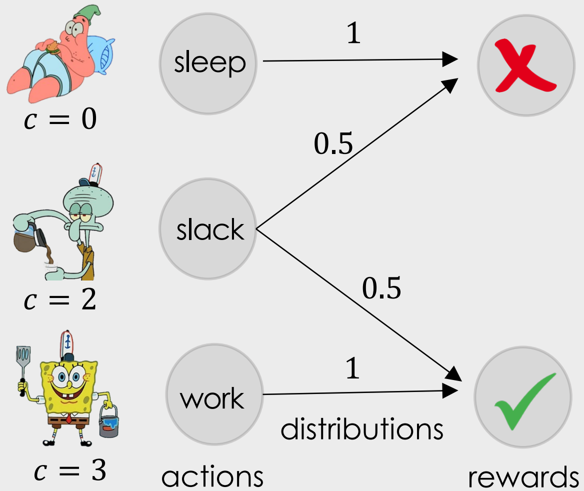

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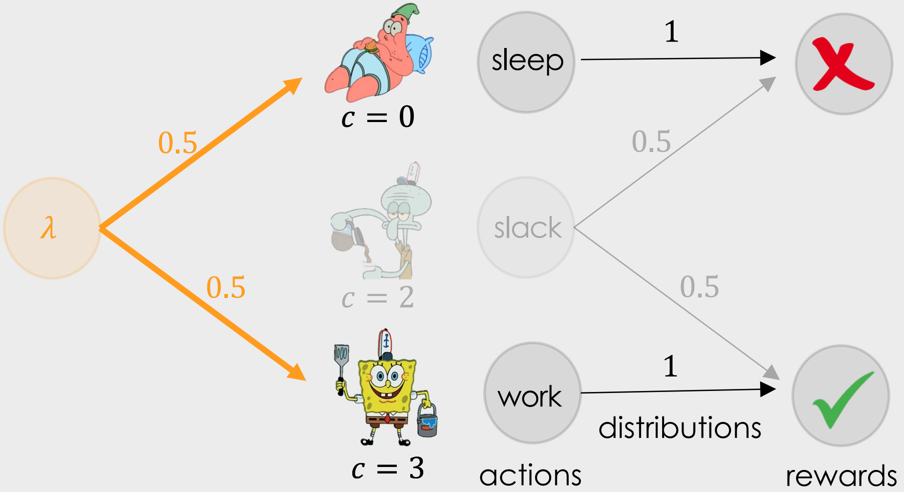

is not implementable

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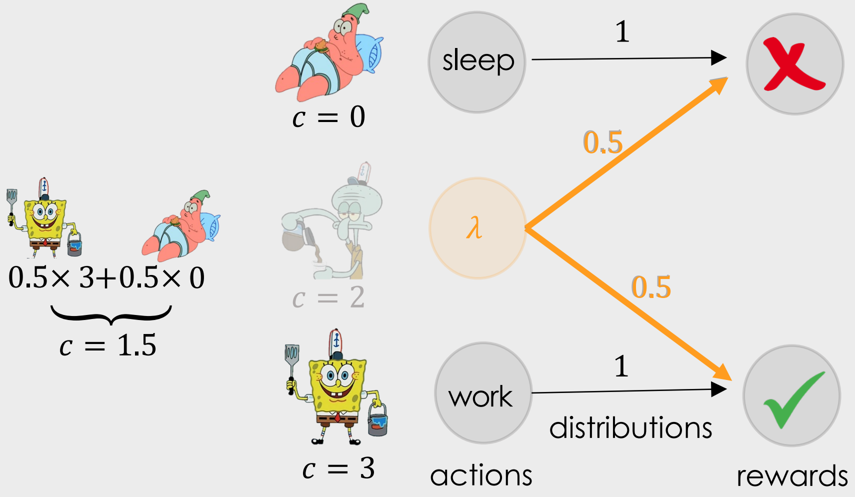
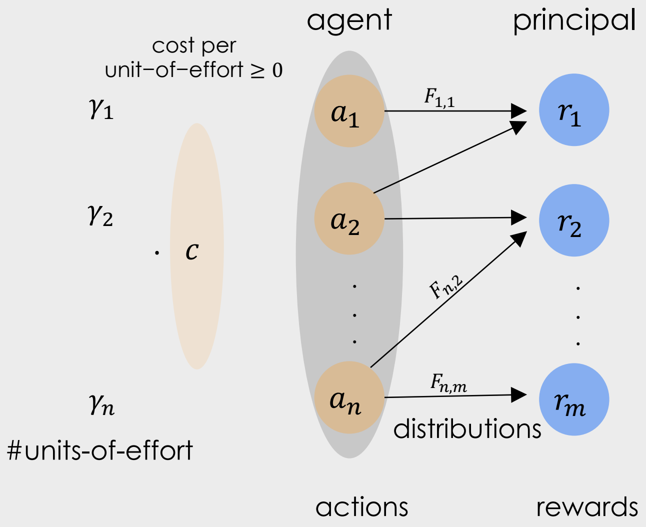


is not implementable

- LP duality approach

Design Space Characterization

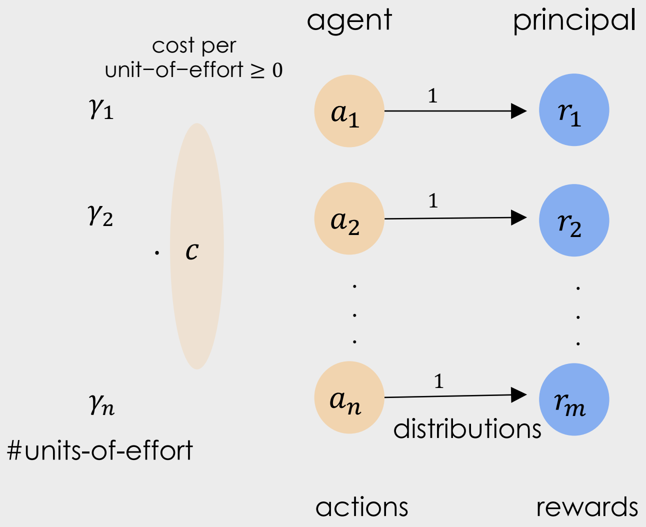
Pure hidden type



Design Space Characterization

Pure hidden type

- x is **implementable** $\Leftrightarrow$ x is **monotone** [Myerson 1981]



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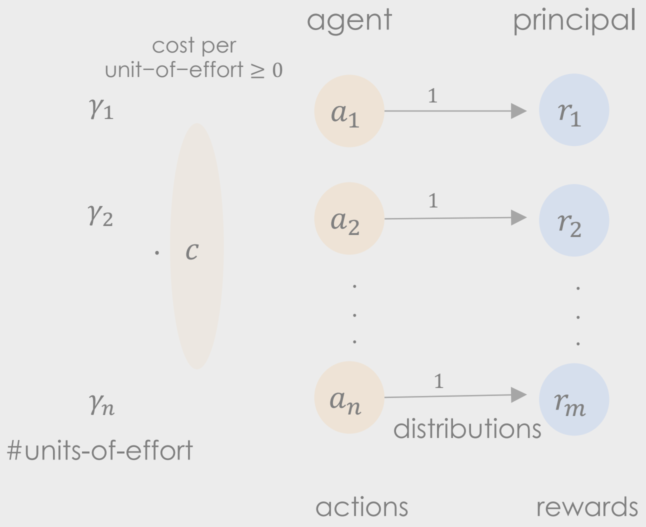
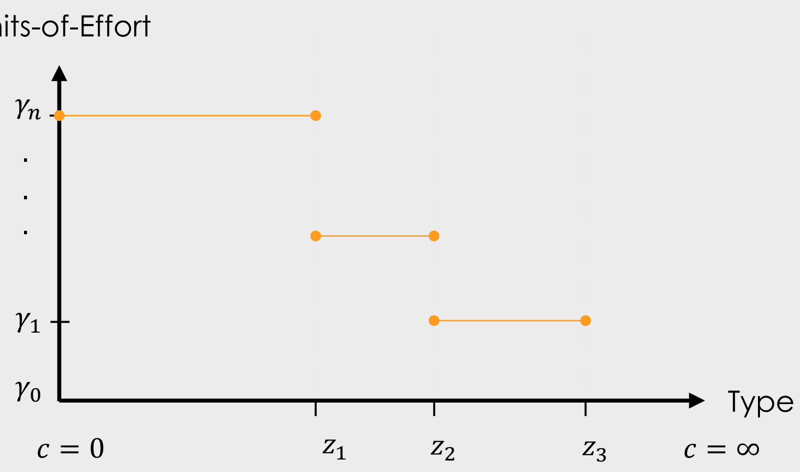
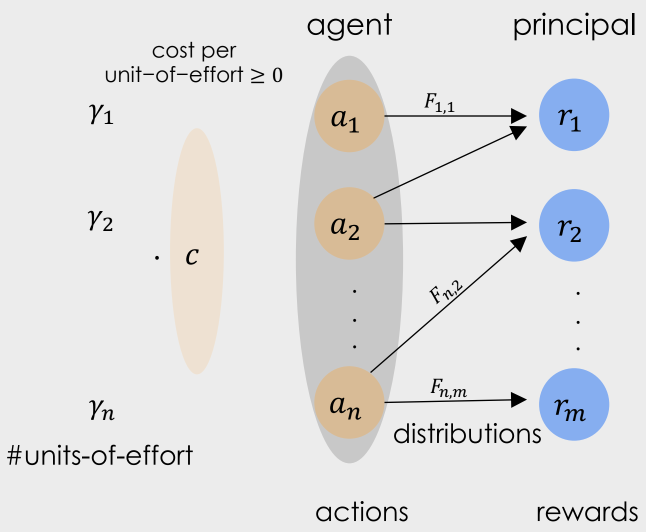


Monotone allocation rule $x: C \to [n]$



Design Space Characterization

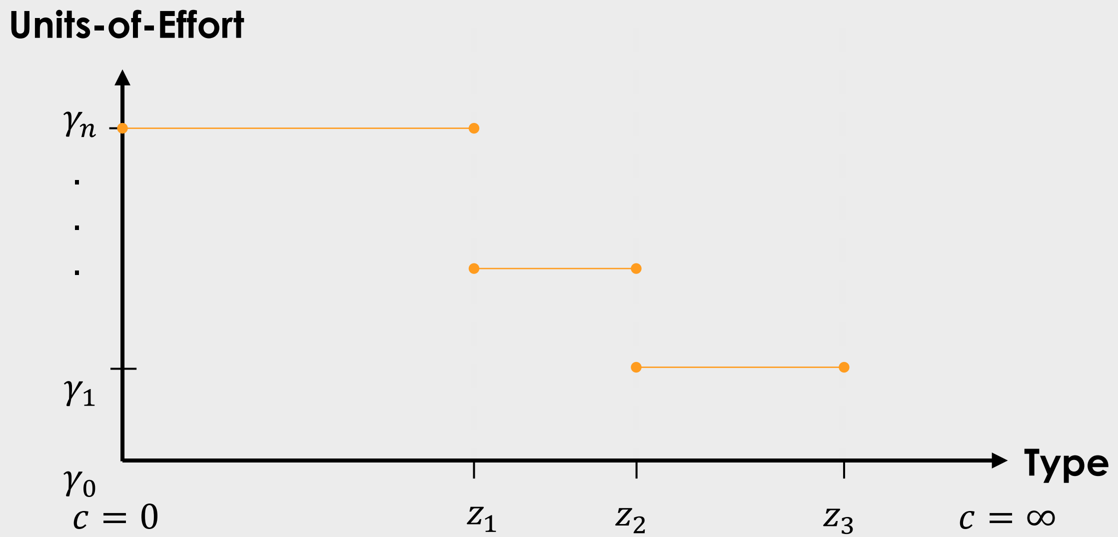
Hidden type and hidden action



Design Space Characterization

Hidden type and hidden action

Proposition [ADT EC'21]. If x is implementable, it is **monotone**



Monotone piecewise-constant allocation rule $x: C \to [n]$

Design Space Characterization

Theorem [ADT EC'21]. x **implementable** $\Leftrightarrow$ exists no **deviation scheme** $\lambda_{(z,k)}$ s.t.

(1) dominant sum of distributions $\sum_{z,k} \lambda_{(z,k)} F_k \geq \sum_z F_{x(z)}$

(2) strictly lower joint cost $\sum_{z,k} \lambda_{(z,k)} \gamma_k z < \sum_z \gamma_{x(z)} z$

Corollary [ADT EC'21]. Optimal contract is **polytime computable** for const #actions

Hardness for constant #actions in the multi-parameter model [Guruganesh-Schneider-Wang'21]

Example

Theorem [ADT EC'21]. x **implementable** $\Leftrightarrow$ exists no **deviation scheme** $\lambda_{(z,k)}$ s.t.

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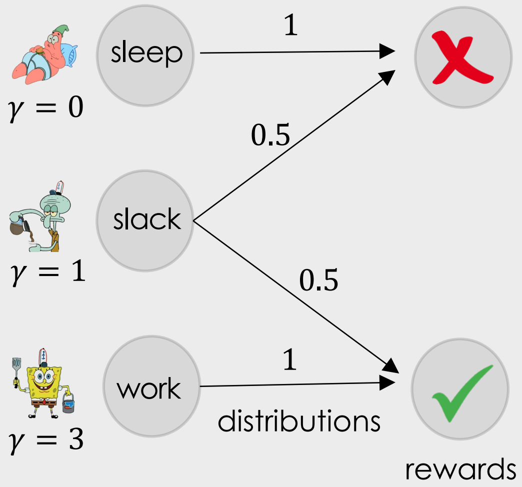

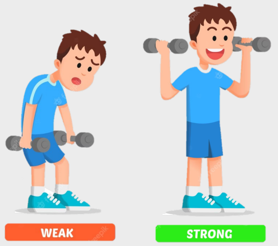

$c \to \infty$ $c = 0$

Example

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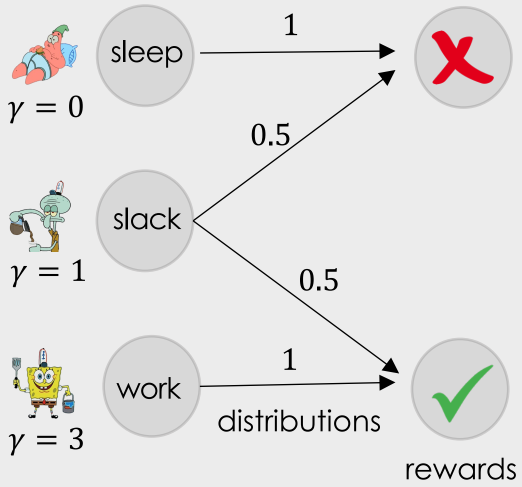

allocation rule: 
$c \to \infty$, $c = 0$ $\to$ $\gamma = 1$

(1) distributions sum (1,1)

(2) joint cost ∞

Example

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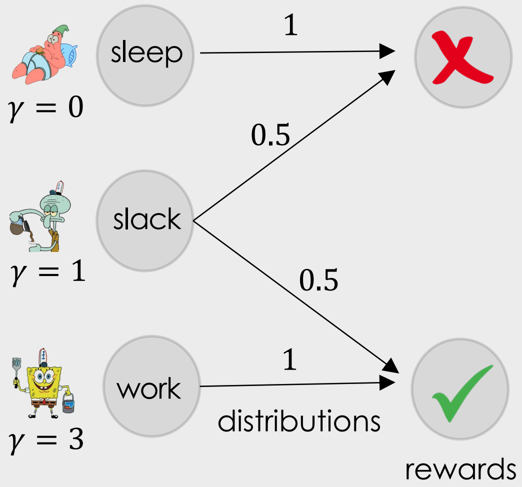
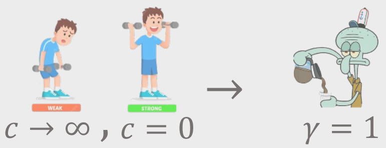


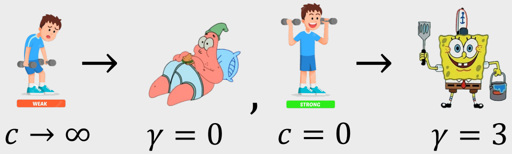
allocation rule:



(1) distributions sum (1,1)

(2) joint cost ∞

deviation:



(1) distributions sum (1,1)

(2) joint cost 0

Optimal Contracts and Their Issues

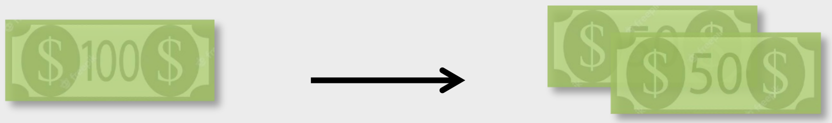
- Informational requirements, extensive analysis ,etc.
- Unintuitive, e.g., non-monotonicity in rewards [DRT EC'19]

Theorem [ADLT EC'23]. In the single dimensional typed model

- Large **menu-size complexity**
- Revenue **non-monotonicity** w.r.t type distribution

Simple Contracts

- In a **linear contract**, the principal offers a fixed share $\alpha \in [0,1]$ of the rewards



100$

50$ to agent
50$ to principal

"It is probably **the great robustness of linear rules** based on aggregates **that accounts for their popularity**. That point is not made as effectively as we would like by our model; we suspect that it cannot be made effectively in any traditional Bayesian model." [Milgrom and Holmstrom 1987]

Simple Contracts

- In a **linear contract**, the principal offers a fixed share $\alpha \in [0,1]$ of the rewards

- **Robustness of linear contracts.** Carroll (2015), Dütting et al. (2019), Yu and Kong (2020), Dai and Toikka (2022), Walton and Carroll (2022)
- **Approximation of linear contracts.** Dütting et al. (2019), Castiglioni et al. (2021), Guruganesh et al. (2021)

Near-Optimality of Linear Contracts

- $\theta(n)$ separation for **point-mass** distributions [DRT EC'19]
  - Boundary case
- **Approximately optimal** with **sufficient uncertainty**
  - The small-tail assumption

principal-agent instances

Not small-tail	Small-tail
Point-mass like	**Sufficient uncertainty**
[Dütting et al. 2019]	**Linear contracts are (near)-optimal!**

Near-Optimality of Linear Contracts

Theorem [ADLT EC'23]. **Revenue** benchmark:

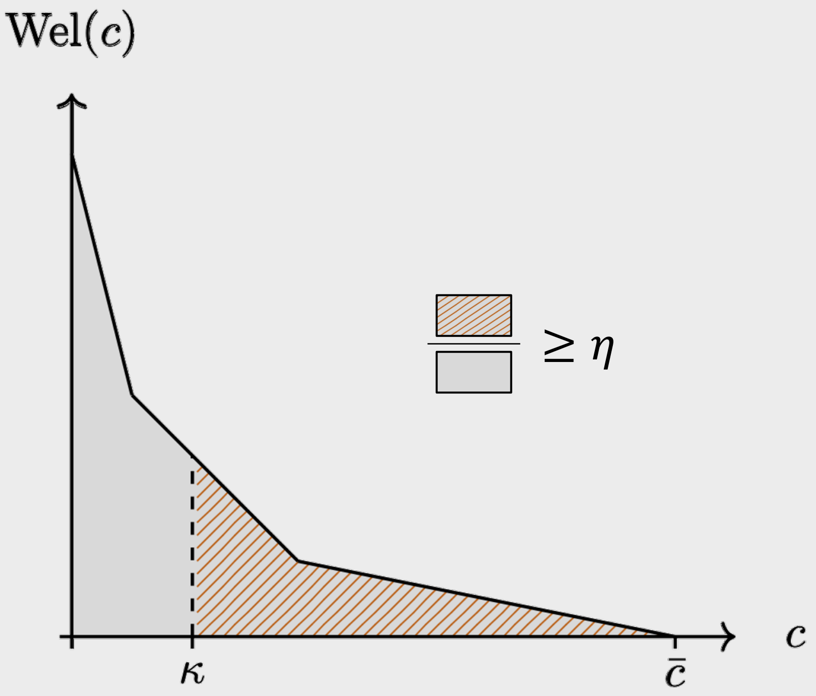
- **3-approximation** for **normal** $\mathcal{N}(\mu, \sigma^2)$ truncated at $c = 0$ with $\sigma \geq 5\eta/2\sqrt{2}$
- **2-approximation** for **uniform** $U[0, \bar{c}]$
  - **Optimal** when $i^*(r, \bar{c}) = 0$
- **2-approximation** for **decreasing densities** (e.g., exponential)

- Constant approximation w.r.t optimal welfare benchmark [ADLT EC'23]

The Small-Tail Assumption

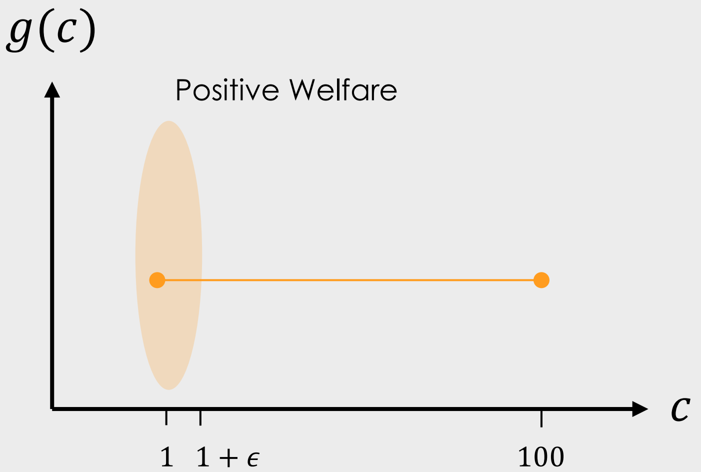
Definition [ADLT EC'23]. Let $\kappa \in [\underline{c}, \bar{c}]$, $\eta \in [0,1]$.

An instance is **(κ, η)-small-tail** if $\text{Wel}_{[\kappa,\bar{c}]} \geq \eta \text{Wel}_{[\underline{c},\bar{c}]}$

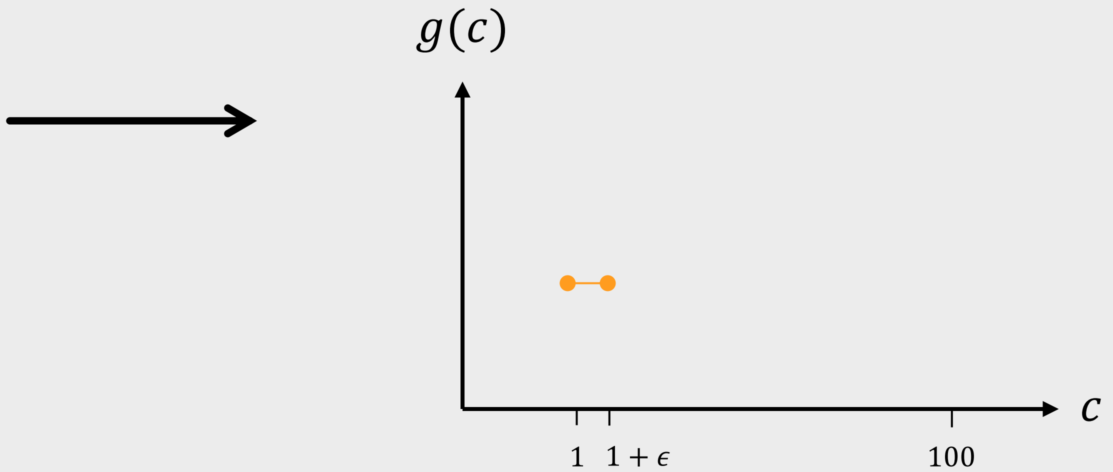


The Small-Tail Assumption

Depends on the entire principal-agent setting



High Uncertainty



Low Uncertainty

Dütting et al. (2019)

Universal Approximation Guarantee

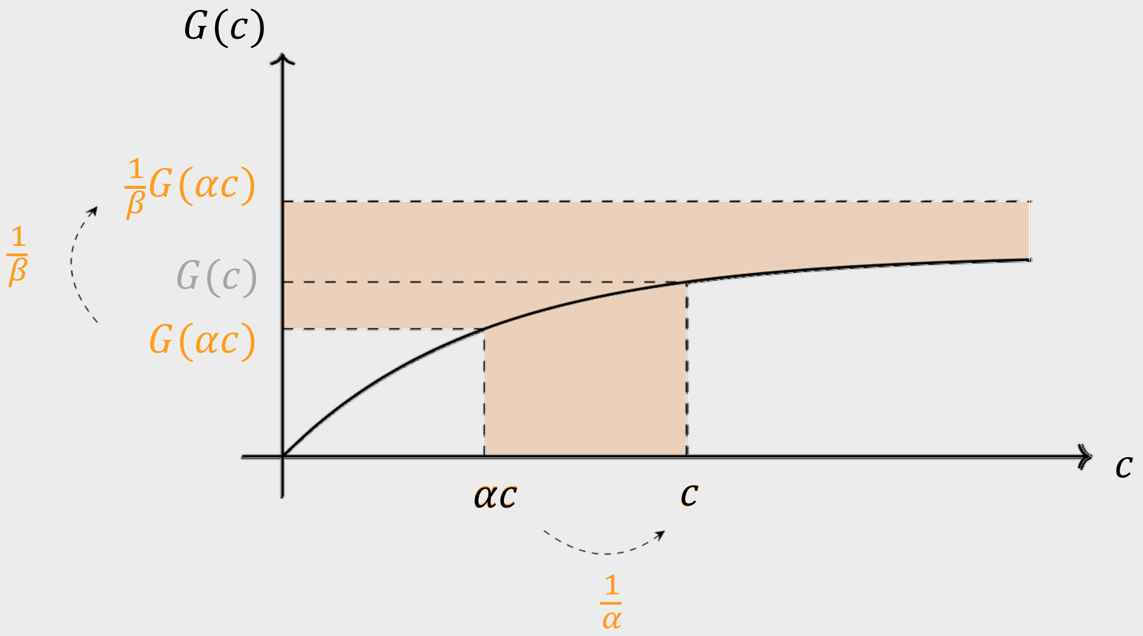
Theorem [ADLT EC'23]. Let $q \in (0,1)$ and $G(c_q) = q$. If for $\alpha, \eta \in (0,1)$ the settings is $(\frac{c_q}{\alpha}, \eta)$-small-tail then linear contract α is at least $(1-\alpha)\eta q$ of the **optimal welfare**

Slowly-Increasing Distributions

- Applies to any CDF and captures its **rate of increase**
- Parametric **approximation** of linear contracts
- Any distribution is slowly-increasing for **some parameters**

Slowly-Increasing Distributions

Definition [ADLT EC'23]. Let $\alpha, \beta \in (0,1)$, and $\kappa \in [\underline{c}, \bar{c}]$. A distribution G is $(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\kappa})$-**slowly-increasing** if $G(c) \leq \frac{1}{\beta} G(\alpha c)\ \forall \kappa \leq c$



Approximation for Slowly Increasing

Theorem [ADLT EC'23]. Let $\alpha, \beta, \eta \in (0,1)$, and $\kappa \in [\underline{c}/\alpha, \bar{c}]$.

Under $(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\kappa})$**-slowly-increasing** and $(\boldsymbol{\kappa}, \boldsymbol{\eta})$**-small-tail**

linear contract α is $(\mathbf{1} - \boldsymbol{\alpha})\boldsymbol{\beta\eta}$ of the **optimal welfare**

Proof Idea for Slowly Increasing

Step 1. Revenue of linear contract α is at least $1-\alpha$ of its welfare

revenue of linear contract		welfare of linear contract
$\mathbb{E}_{c\sim G}[(1-\alpha)R_{x(c)}]$	$\geq \mathbf{(1-\alpha)} \times$	$\mathbb{E}_{c\sim G}[R_{x(c)} - \gamma_{x(c)}c]$

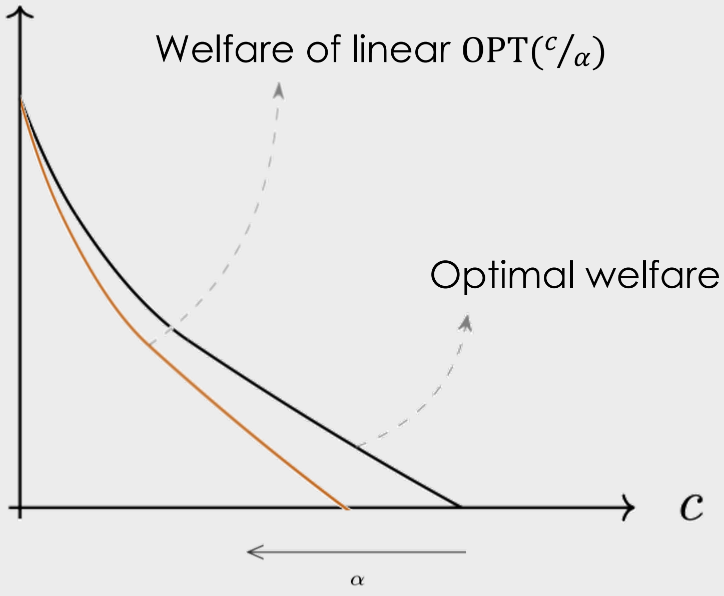
linear contract revenue $\geq (\mathbf{1}-\boldsymbol{\alpha}) \times$ **linear contract welfare** $\geq (1-\alpha) \times \beta \times$ optimal welfare

Proof Idea for Slowly Increasing

Step 2. welfare of linear contract α is at least β of the optimal welfare

- When maximizing welfare, the agent maximizes $R_i - \gamma_i c$
- For linear contract α:

  The agent maximizes $\alpha R_i - \gamma_i c$ which is $R_i - \gamma_i \frac{c}{\alpha}$



linear contract revenue $\geq (1-\alpha) \times$ **linear contract welfare** $\geq (1-\alpha) \times \boldsymbol{\beta} \times$ **optimal welfare**

Summary and Future Directions

- **Single-parameter** model of types
- **Characterization** of the design space
- Counter-intuitive and **undesirable** properties of **optimal contracts**
- **Linear contracts** are near-optimal

Future directions:

- Other **forms of simple contracts** that are near-optimal
- Contracts that involve **multiple agents**
- **Applications** of this theory

Thank You!

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